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A RE-EVALUATION OF ZERO PRESSURE GRADIENT COMPRESSIBLE TURBULENT BOUNDARY LAYER MEASUREMENTS

James E. Danberg

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

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13. ABSTRACT

A number of compressible turbulent boundary layer velocity and temperature profiles with zero pressure gradient have been collected and prepared for computer analysis. An assumed equation for these profiles has been chosen allowing four constants to be adjusted by a nonlinear least squares technique to fit the experimental data. The four constants are: a velocity scale, boundary layer thickness, the constant of the semi-log region and the wake constant, Π. This equation is analogous to Cole's incompressible law of the wall and wake but uses a generalized velocity to account for compressibility. Measurements from 45 adiabatic wall tests have been analyzed covering a Mach number range from 2 to 6 and a momentum thickness Reynolds number range from 2.3×10^3 to 7.5×10^5. Of these profiles, 29 included skin friction balance data which allowed direct evaluation of the universal constant of turbulence (mean value of $K = .43$) through comparison between the shear velocity and the profile velocity scale. The constants of the semi-log and the wake region were found to be independent of Reynolds and Mach numbers. A similar analysis was carried out for the limited number of total temperature profiles.

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TABLE OF CONTENTS

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LIST OF ILLUSTRATIONS

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LIST OF SYMBOLS

u = local mean velocity

u_s = characteristic velocity scale for turbulent velocity profile

u^{++} = nondimensional velocity = $\frac{1}{u_s} \int_o^u \sqrt{\frac{\rho}{\rho_w}}\, du$

u_τ = shear velocity = $\sqrt{\tau_w/\rho_w}$

y = distance normal to the surface

C = profile constant associated with semi-logarithmic region

H = local total enthalpy = $C_p T_t$

H_S = characteristic total enthalpy scale for the thermal boundary layer

H^{++} = nondimensional total enthalpy = $\frac{1}{H_s} \int_o^H \sqrt{\frac{\rho}{\rho_w}}\, d(H-H_w)$

T = temperature

α = thermal diffusivity

β = $(T_{aw}-T_w)/(T_{t\delta}-T_w)$

δ_s = velocity boundary layer thickness

Δ = thermal boundary layer thickness

κ = universal constant of turbulence (mixing length constant)

ν = kinematic viscosity

Π = profile constant associated with the wake region

ρ = density

τ = shear stress

ω = wake function

Subscripts

aw = adiabatic wall conditions

s = characteristic scale

t = total

I. INTRODUCTION

Most experimentalists investigating some aspect of compressible turbulent boundary layers have attempted to measure the distribution of mean velocity and temperature through the layer. During the past 20 years, a considerable number of these profile measurements have been reported. It would appear that a sufficient body of data is now available to begin a more systematic correlation approach with the objective of trying to obtain more quantitative information from the boundary layer surveys themselves, rather than making comparison between theory and skin friction, heat transfer or other surface data. The approach of this study is an extension to compressible flow of some of the tests used by Coles and Hirst[1*] to "classify and criticize" the available incompressible data for the AFOSR-IFP-Stanford Conference on Turbulent Boundary Layers.

II. APPROACH

In order to exploit the above proposal, the following approach was adopted:

(a) Samples of published compressible turbulent boundary layer survey data were collected and stored on IBM cards. The initial search turned up about 150 zero pressure gradient profiles where tabulated data were available or graphical data could be reasonably evaluated. These data cover a range of Mach numbers from 1.5 to 12 and momentum thickness Reynolds numbers from 10^3 to 10^6. Only perfect gas cases were considered with air or nitrogen as the test medium. The geometry of the test surfaces were mostly flat plates and nozzle walls where the pressure gradient effects are expected to be small. The main results reported here will be concerned with adiabatic wall conditions which limits the range of Mach numbers from 2 to 6.

Obviously, not all of the surveys considered are equal in quality and part of the evaluation procedure must be concerned with the determination of internal consistency and consistency between surveys. It is also evident where more experimental data are needed. In general, it may be concluded that none of the experimenters used all the techniques available to them - especially the investigators who concern themselves with skin friction balance measurements in zero pressure gradient adiabatic wall boundary layers. The most notable omission of these investigators was temperature profile measurements. Thus, their important measurements have to be interpreted using theory or correlations based on other temperature measurements.

(b) An analytical framework was assumed in order to reduce the mass of data points into a manageable set of numbers from which to draw some

*References are listed on page 27.

conclusions. The equations chosen to represent the velocity and temperature distribution are not final recommendations, but they do represent a first step, to which modifications may be introduced as required or alternative approaches adopted and then tested against the available experimental evidence. The initial approach is based on similarity concepts as extensions of the law of the wall - the law of the wake suggested by Millikan[2], and Coles[3] and many others for the compressible turbulent boundary layer. The procedure makes the following assumptions:

(1) The effects of compressibility are accounted for by forming a reduced velocity

$$u' = \int_0^y \sqrt{\rho/\rho_w}\, du. \tag{1}$$

This assumption is quite arbitrary in the present context although it is consistent with the Prandtl mixing length approach as applied to compressible flow by VanDriest[4], Moore[5], Spalding and Chi[6] among many others. The assumption can be tested to some extent by trying other alternative assumptions.

(2) The boundary layer consists of two basic regions, a wall region and a wake or defect region describable by functions of two essentially independent variables. In the wall region, it is assumed that the velocity distribution can be described in terms of a velocity scale (u_s) and a length scale ν_w/u_s where u_s is to be determined from experimental velocity profile data. In the defect region, the same velocity scale is assumed to apply but a new length scale δ_s characteristic of the total boundary layer thickness is assumed, δ_s is also to be evaluated from experimental data. The specific definition of δ_s depends on the assumed form of the wake function.

(3) The two regions of the boundary layer are connected by a region of overlap where formulas for both regions predict the same velocity distribution. Millikan[2] has shown that this implies that the velocity is a semi-logarithmic function of y in the overlap region.

$$\frac{u'}{u_s} = \ln\left(\frac{y}{\nu_w/u_s}\right) + C$$

or: (2)

$$\frac{u'}{u_s} = \ln\left(\frac{y}{\delta_s}\right) + D.$$

Note that the above semi-log equations are nearly identical with the usual turbulent boundary layer semi-log equations. The one difference is that the mixing length constant, κ, does not appear and may be considered to have been absorbed into the velocity scale, u_s. If u_s is replaced by u_τ/κ and C by $\kappa C + \ln\kappa$, then the usual form of this relation is recovered. The reason for this new definition of the velocity scale

is because κ and u_s cannot be determined independently from experimental velocity profile data. Another reason for choosing this definition is that it clearly brings out that κ^2 is the slope of the non-dimensional velocity profile at $y = 0$; that is, if the wall length scale is valid at $y = 0$, then:

$$u' = u_s \, f(u_s y/\nu_w)$$

and assuming Newtonian friction at the wall

$$\tau_w/\rho_w = \nu_w \frac{du'}{dy}\bigg|_{y=0} = u_s^2 f'(0)$$

with the result that

$$\kappa^2 = f'(0) \; . \tag{3}$$

Since the probe data is relatively poor near the wall in the laminar sub-layer, it is necessary to use skin friction balance measurements to obtain the relationship between u_s and u_τ and the ratio of u_τ to u_s provides a method of evaluating κ.

The equation used to describe the compressible turbulent velocity profile (neglecting the laminar sublayer) is obtained by analogy with the incompressible flow equation proposed by Coles.

$$u^{++} = \frac{1}{u_s} \int_0^u \sqrt{\rho/\rho_w} \, du' = \ln\left(\frac{u_s y}{\nu_w}\right) + C_u + 2\Pi_u \, \omega(y/\delta_s) \tag{4}$$

where approximately

$$\omega(y/\delta_s) = \sin^2\left(\frac{\pi}{2}\frac{y}{\delta_s}\right) . \tag{5}$$

This equation is assumed valid except in the laminar sub-layer and beyond the point where

$$\frac{du^{++}}{d(y/\delta_s)} = 0 \; .$$

The four profile constants u_s, δ_s, C_u and Π_u are determined by a least squares fit to each measured velocity profile.

III. TEMPERATURE PROFILE

The temperature distribution was calculated for many surveys where temperature measurements were not available by using a well known modification of the Crocco temperature-velocity relationship:

$$\frac{T_t - T_w}{T_{t\delta} - T_w} = \beta \frac{u}{u_\delta} + (1-\beta)\left(\frac{u}{u_\delta}\right)^2 \tag{6}$$

where

$$\beta = (T_{aw} - T_w)/(T_{t\delta} - T_w) \ .$$

As has been noted by many investigators, almost any expression, similar to the above, gives adequate results for low Mach number data under adiabatic conditions. Across most of the layer, the total enthalpy is nearly constant in any case. However, for the high heat transfer or high Mach number situation, the deviation of experimental data from the Crocco relationship is known to be significant.

There appears to be a need to find better methods of correlating temperature profile data than the Crocco equation. In analogy to the above method of describing the velocity profile, two length scales (δ_H, $\alpha_w/\sqrt{H_s}$) and a total enthalpy scale (H_s) are introduced in an attempt to find a more suitable correlation formula. The resulting equation is:

$$H^{++} = \frac{1}{H_s} \int_0^H \sqrt{\frac{\rho}{\rho_w}}\, d\,(H-H_w) = \ln \frac{H_s y}{\alpha_w} + C_H + 2\Pi_H \omega_H \tag{7}$$

where $\omega_H \cong \sin^2 \frac{\pi}{2} \frac{y}{\Delta}$

H_s = characteristic enthalpy scale

Δ = thermal layer characteristic length scale (not necessarily equal to the thickness of the thermal layer)

The enthalpy scale and the thermal boundary layer thickness and the two profile constants (C_H and Π_H) are obtained from each profile where temperature data are reported by the same least squares method used for the velocity profile.

A related approach for the incompressible enthalpy profiles has been explored by Alber and Coats[7] in a recent paper. Their analysis is based on a mixing length analysis where it is assumed that H_s is proportional to the wall heat transfer rate and thus $H_s = 0$ for the adiabatic case. In the adiabatic compressible boundary layer there is a characteristic enthalpy distribution which has many of the same features of wall and wake regions as the velocity profile; thus, a finite enthalpy scale can be defined which is valid over a significant part of the enthalpy distribution although it does not appear that this similarity can extend to the wall as seems to be the case for the velocity profiles.

IV. LEAST SQUARES METHOD

An initial computer program was written to determine u_s, δ_s, C_u and Π_u for the velocity profile and the corresponding quantities for the temperature profiles, where available, by an iteration procedure using the following equation.

$$u = a_1 \ln a_2 y + a_3 + a_4 \sin^2 a_2 y \tag{8}$$

where a_1, a_2, a_3, and a_4 can be used to calculate the more meaningful profile quantities. Since δ_s (or a_2) is contained within the transcendental function, it cannot be evaluated by the usual methods and, therefore, its value was first estimated and the other three constants determined in a straight-forward way. The computer program calculated the standard deviation of the data points with respect to the resultant equation. A number of such calculations were required to bracket the minimum value of the standard deviation to a predetermined degree of accuracy. Later, a general weighted nonlinear least square fit subroutine, obtained from the U. S. Naval Ordnance Laboratory considerably shortened the iteration procedure with a higher degree of accuracy. This program computes the four constants from four initial estimates by assuming small perturbations to the a's which allows the equation to be expressed by the linear terms of a Taylor expansion whereby an improved estimate of the a's is obtained to start a new iteration. Again, a predetermined test is applied to terminate the calculation.

There are two problems that arise in the least square fitting procedure. First, the equation is quite nonlinear with a number of "local" maximums and minimums in the standard deviation. This was a particularly troublesome problem with the first technique used. For example, if δ_s is overestimated, a_2 is small and the best fit equation tends toward the situation where the sine-squared term is eliminated. Trial and error changes of the initial values of the constants usually corrected the difficulty which was detected by finding absurd values for the a's or by the poor fit obtained when the data were graphed.

The second problem has to do with a priori unknown range of validity of the basic equation. As already has been pointed out, the equation used here cannot be used to describe either the sublayer or the free stream. Unfortunately, it is often difficult to identify the data points which should be excluded from the calculation except by inspection of the plotted results. Ultimately, part of this problem can be eliminated by designing the form of the velocity profile which is valid across the entire boundary layer although there may still be a need to exercise judgment about those points effected by wall interference.

V. DISCUSSION OF RESULTS

Figures 1[19] and 2[9] show representative non-dimensional velocity profiles of u^{++} versus $u_s y/\nu_w$. The solid line drawn through the data is the best fit to Eq.(4). The standard deviation in u^{++} is .077 and .022, respectively, which can be interpreted as .6 and .25 percent of the free stream velocity. This shows that Eq. (4) describes the velocity data in the range where it has been applied to the same order of accuracy as that quoted for current pressure transducers.

Figure 3 shows an example of the quality of the fit of the $\sin^2$ term for the wake function of Coles. As pointed out earlier, no data beyond $y = \delta_s$ has been included in the fitting process.

A. Velocity Scale

As previously noted, if the similarity of the velocity profile is valid to $y = 0$ then the velocity scale obtained from the boundary layer profile is related to the shear velocity through Newton's law. This hypothesis is tested by a comparison between u_s and u_τ where values of u_τ have been obtained from skin friction measurements (Ref. 8 to 11). Figure 4 shows the result in non-dimensional terms of u_δ/u_s versus u_δ/u_τ. The mean line through the data has a slope of .43 which is in reasonable agreement with the most frequently quoted values of .4 or .41 for the universal constant, κ.

B. Profile Constants

The computer evaluation of the constants C_u and Π_u are given in Figure 5 including results from 45 profiles obtained from 12 investigators (Ref. 8 to 19). The constants are plotted versus the momentum thickness Reynolds number and no discernible trend is evident although there is considerable scatter in the data about the mean values of $C_u = 1.77$ and $\Pi_u = .81$. Also, there is no observable trend of C_u or Π_u with Mach number in the range of 2 to 6 or with the geometry of the test surface (flat plate and nozzle wall data are included).

C. Total Temperature Profile

Figures 6 and 7 illustrate the ability of the assumed equation to fit the total enthalpy distribution for one adiabatic wall case of Sturek[19]. The solid line through the data is best fit to Eq. (7) with the data points at both extremes omitted which obviously do not agree with the trend of the equation. The wake function is approximated by the $\sin^2$ term equally as well as in the velocity case. A number of points beyond $y = \Delta$ fall on the computed line although there is a region at the outer edge of the profile where the transition to the constant free stream temperature occurs that cannot be included. Figure 6 also shows a sharp departure from Eq. (7) that occurs near the wall for these adiabatic heat transfer cases.

D. Temperature Profile Constants

Profile constants from only ten temperature surveys at near adiabatic wall conditions have been tested and eight are at essentially the same conditions for the present purposes. These preliminary results can thus be summarized in the following table where the eight profiles of Sturek[19] have been replaced by their average values.

Temperature profile constants from best fit to Eq. (7)

Ref.	M_δ	Π_H	C_H	$Pr_w^2 H_s/u_s^2$	$\frac{\Delta}{\delta_s}$
12	2.49	.686	-3.74	.89	1.04
14	5.92	.902	- .82	.56	.68
19	3.5	.932	-5.32	.69	.83

These results represent such a small sample that the specific values should be considered highly preliminary; however, it is interesting that for the adiabatic wall conditions, H_s is approximately the same as $(u_s/Pr_w)^2$ and thus $\sqrt{H_s}y/\alpha_w \approx u_s y/\nu_w$. Some initial results which include the effects of heat transfer show that H_s is strongly affected by the heat transfer rate and increases with increasing heat transfer. The relatively large negative values of C_H can be associated with the size of the thermal sublayer in which the total enthalpy distribution is dominated by conditions at the wall. The thermal sublayer is an order of magnitude larger than the velocity sublayer as can be seen by comparing Figures 1 and 6. The wake constant Π_H is about the same magnitude as the velocity parameter. The thickness parameter Δ is generally smaller as might be expected although the total thermal boundary layer may be up to twice as large as Δ and therefore, larger than the velocity boundary layer.

VI. CONCLUSIONS

A number of velocity and temperature profile measurements for compressible (Mach number 2 to 6) turbulent boundary layers (momentum thickness Reynolds numbers = 2.3×10^3 to 7.5×10^5) have been re-evaluated in terms of similarity concepts developed in incompressible flow. The velocity (or enthalpy) profile equation with four adjustable constants has been shown to adequately describe the velocity (or total enthalpy) with a high degree of accuracy except in the sublayer and in a transition region near the free stream. The velocity scale paremeter (u_s) is shown to be proportional to the shear velocity (u_τ) and the constant of proportionality equals .43 which is in reasonable agreement with the

incompressible values. The profile constants C_u and Π_u have been found to be independent of Reynolds number and Mach number although there is considerable scatter in the data. Preliminary data has shown that the total temperature profile can be described in a procedure analogous to that used for the velocity profile and that the enthalpy scale is approximately the same order as the square of the velocity scale. The thermal sublayer for the cases investigated was found to be considerably larger than the velocity sublayer.

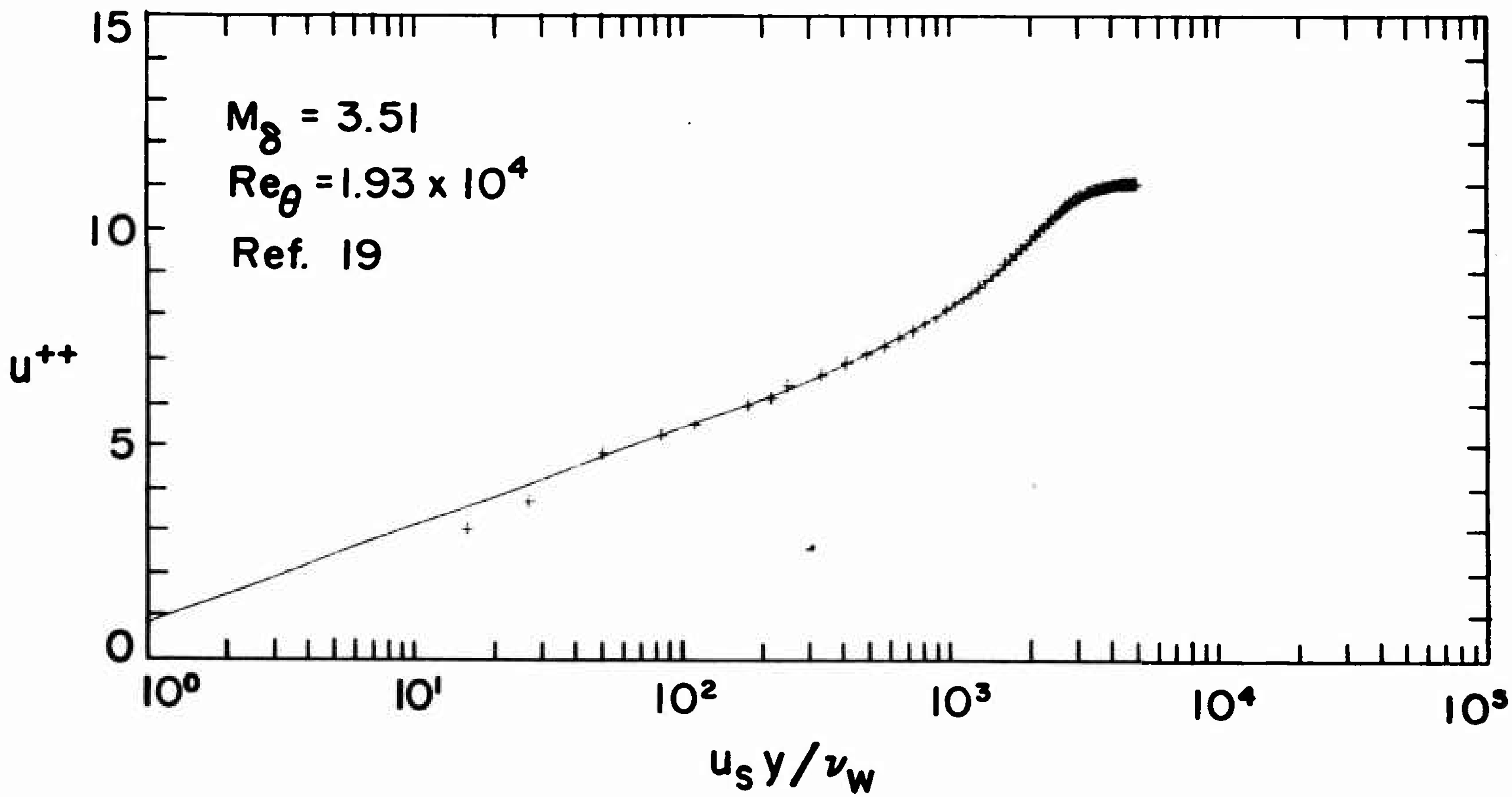

Figure 1. Non-Dimensional Velocity Profile

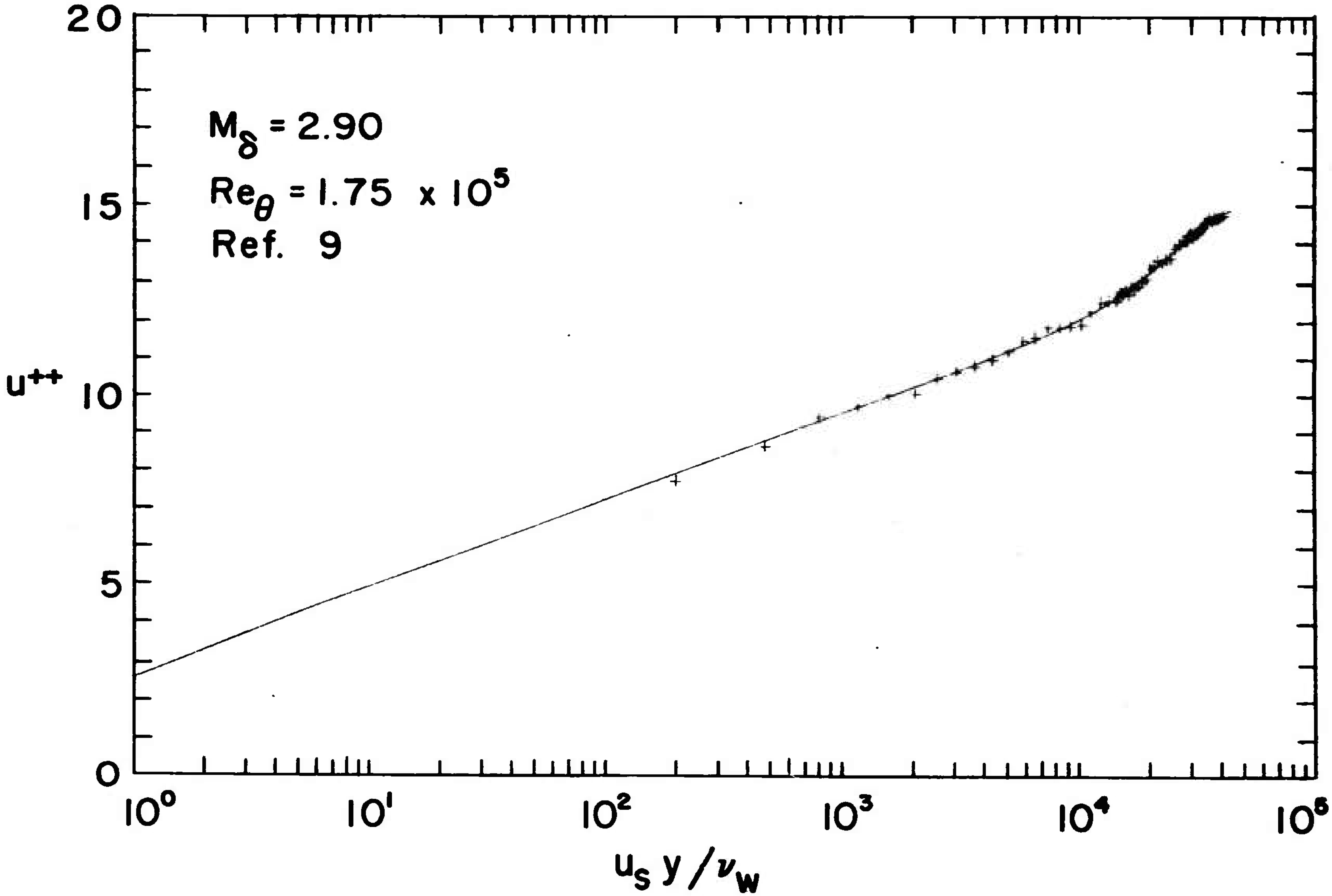

Figure 2. Non-Dimensional Velocity Profile

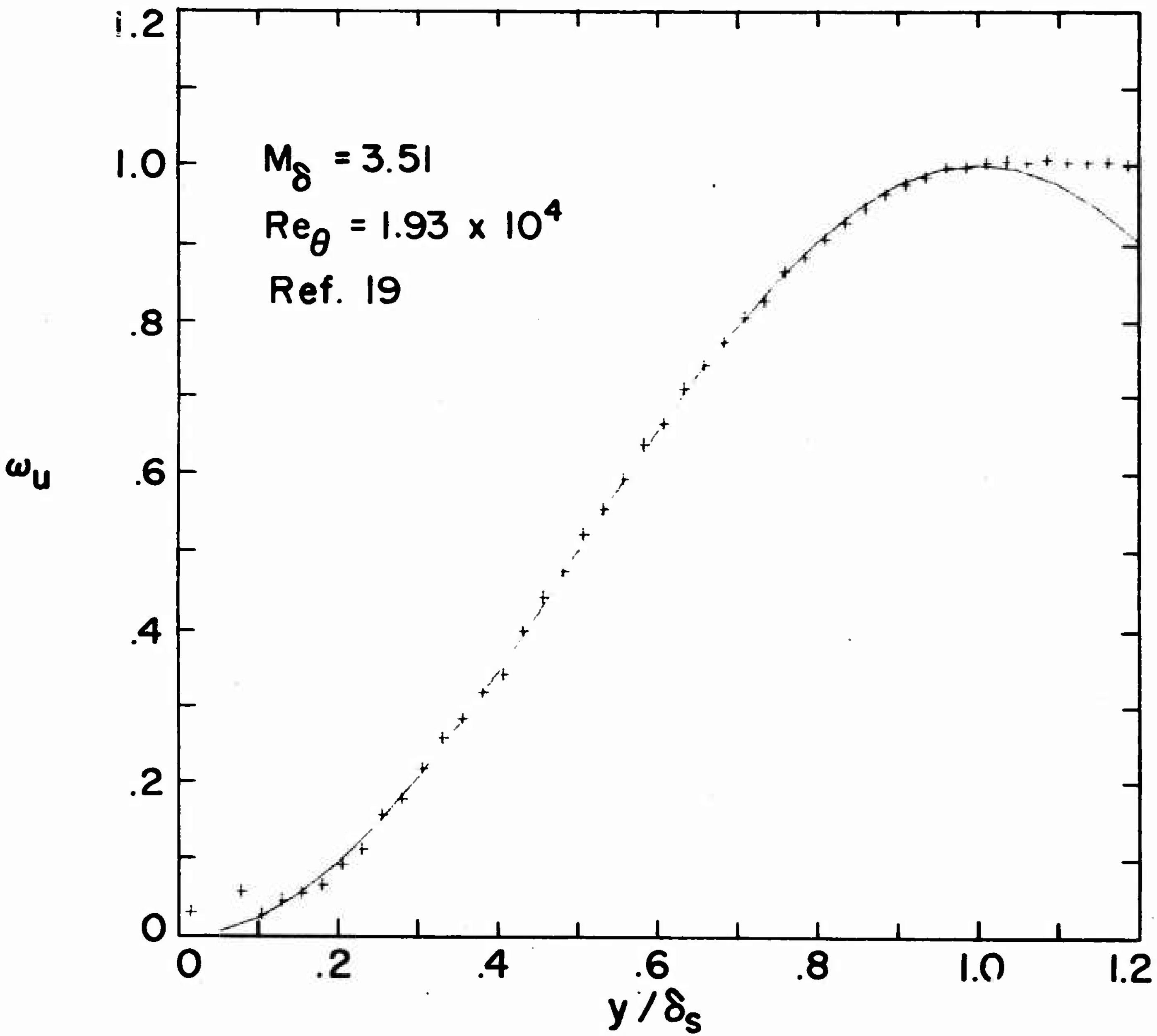

Figure 3. Wake Function for Velocity Scale

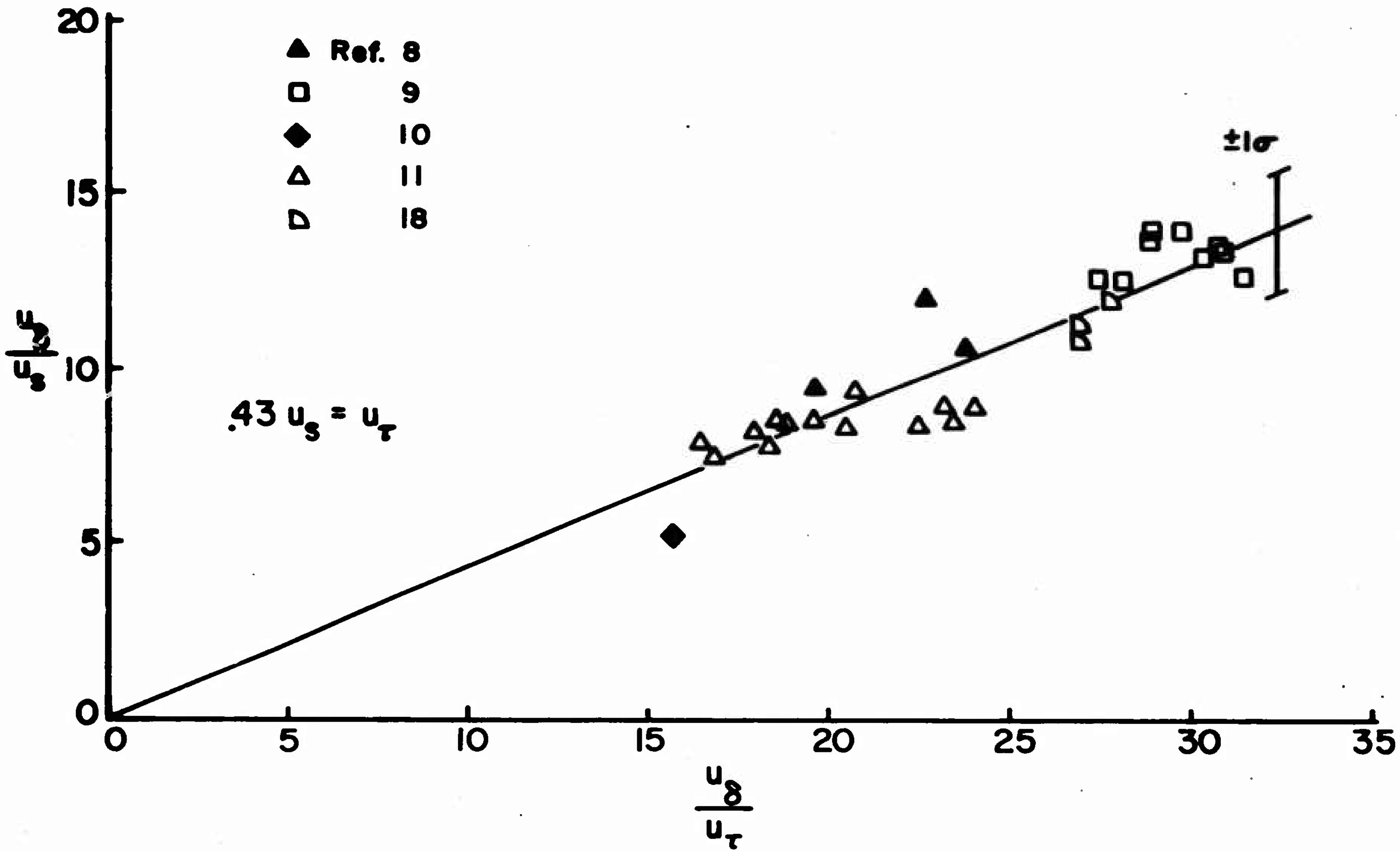

Figure 4. Velocity Scale Parameter versus Shear Velocity

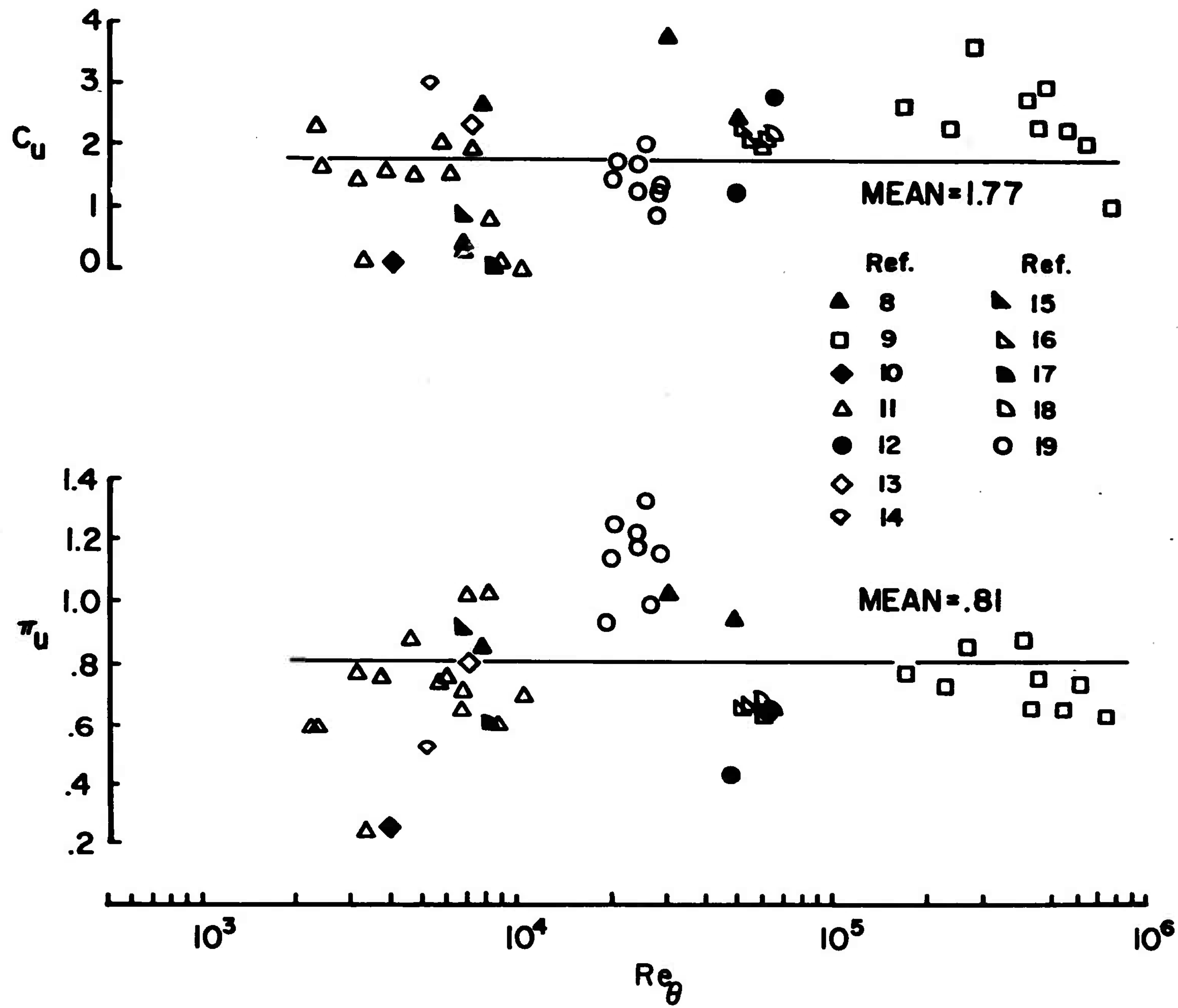

Figure 5. Profile Parameters C_u and Π_u versus Momentum Thickness Reynolds Number

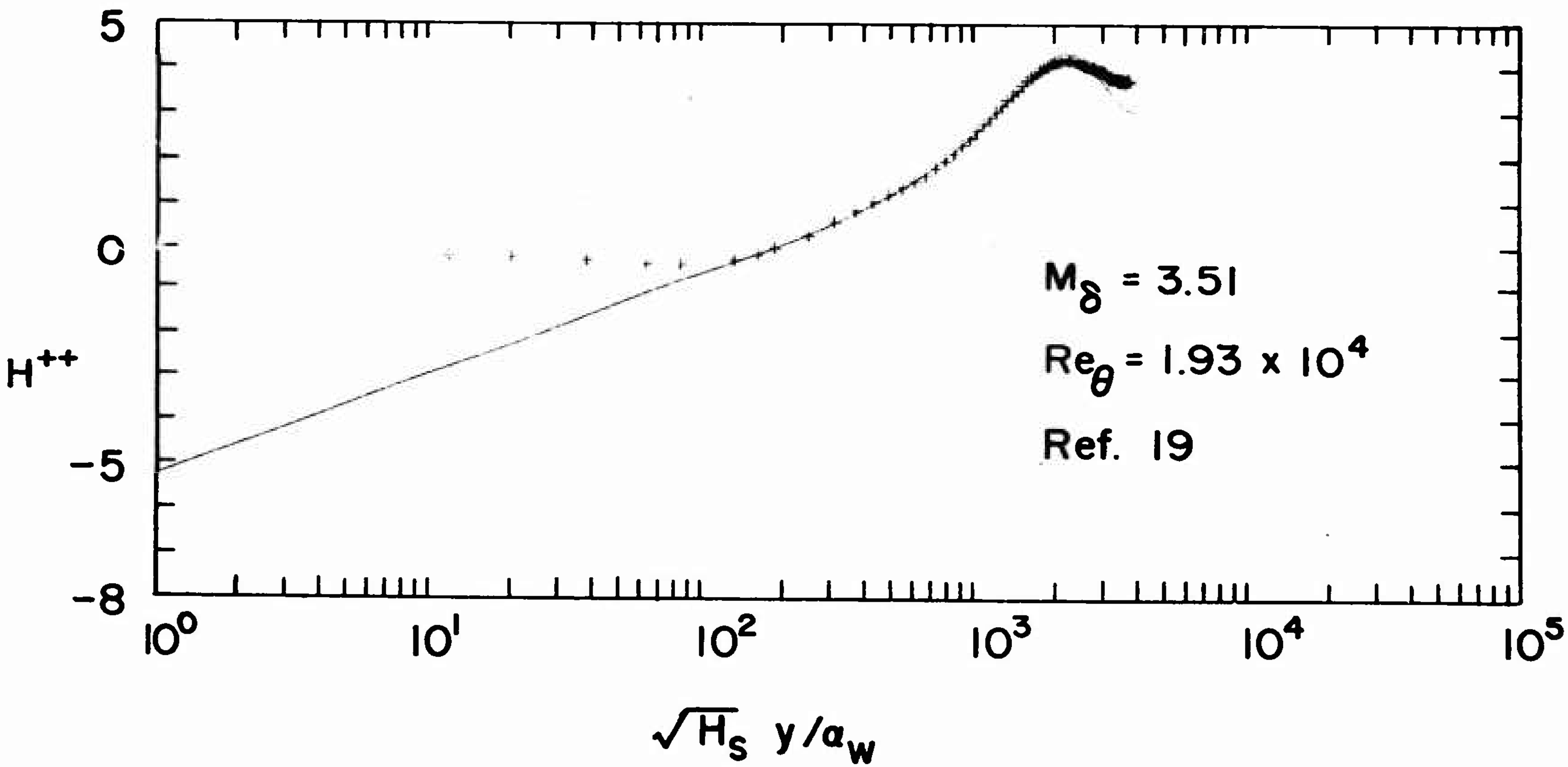

Figure 6. Non-Dimensional Total Enthalpy Profile

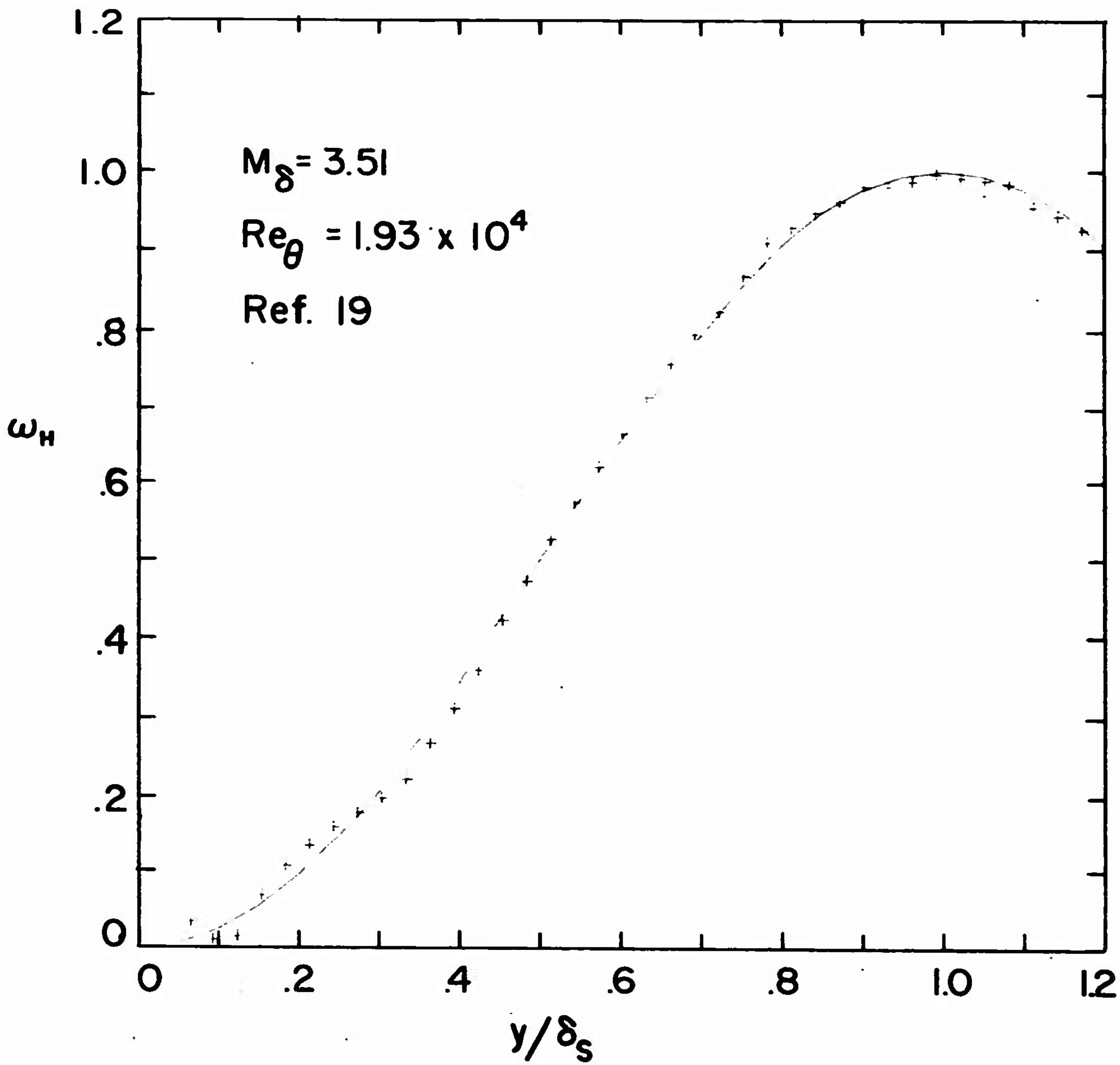

Figure 7. Wake Function for Total Enthalpy Profile

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